LETTERS TO THE EDITOR

Editor's Note: Papers concerned with the settling of concentrated suspensions usually receive very critical comments from reviewers regarding the basic approach (whatever it may be). I have followed a general policy during the past few years of accepting papers which represent controversial points of view in this field and of encouraging discussion in the "Letters to the Editor" column, in the hope that such open discussion will help bring the controversy to a useful conclusion. Dr. Dixon was a reviewer of Prof. Font's paper under discussion here, and I invited Dr. Dixon to submit a "Letter to the Editor" at the time that I accepted the paper.

To the Editor:

Font (Jan., 1990) made an interesting contribution to gravity thickening theory in showing an inequality relationship between instantaneous sediment depth in batch settling and sediment depth required in a corresponding continuous operation. (In the latter, negligible sludge funneling was assumed.) Given that most flocculated sediments can be treated approximately as having a maximum achievable concentration, ϵ_{sb}^{max}, the inequality relationship approaches equality as the bottom concentration in the batch case, and the underflow concentration in the corresponding continuous case approaches ϵ_{sb}^{max}.

If sludge funneling can be neglected in a thickener, minimum area requirements and sludge depth can be calculated from data on the terminal settling velocity (or permeability) and the compressive yield stress of the solids as functions of solids concentration. It seems that sludge funneling is not a negligible factor (Dixon, 1980). Leaving aside this question and assuming vertical plug flow of the sludge, however, the sludge depth is given by an integral expression derived by Fitch (1966). This integral is directly analogous to those used for calculating heights or lengths of packed distillation columns, cooling towers, heat exchangers, etc. It involves operating and equilibrium lines which cannot cross; and the closer they are, the greater the height or length required for the equipment. Batch settling tests are the easiest tests to carry out for obtaining the necessary permeability and compressibility data. The amount of testing required, however, is normally quite extensive, and the central theme of the present paper is obtaining continuous depth requirements from batch results in a more direct manner.

While agreeing with the main conclusion given in the paper, the writer believes the derivation to be seriously in error because of its references to "Case 1." If the concepts put forward are to be properly understood, some comment is necessary.

Before embarking on this, the writer is faced with a difficulty in that Font's treatment makes use of the "$u = u(c)$ assumption" in what he refers to as the "hindered settling" concentration range. (The writer usually uses the term "free-settling" instead; and to avoid confusion the phrase "no-compression" will be used. That is, the concentration range being referred to is where the flocs are not in contact and so not hindered by "compression" effects, in the "suspension," as distinct from the "sediment.") The writer does not accept that $u = u(c)$ can apply when concentration change is occurring, and so does not accept the predictions from this assumption as obtained using the methods demonstrated by Kynch (1952) and employed by Font (Dixon, 1977). However, this is not the question under consideration here.

To simplify the discussion, the case will be considered where the initial concentration in the batch test, ϕ_{so}, is in concentration Region 1, using the Shannon et al. notation, rather than Region 3 as used in the paper. In Region 1, the writer's analysis leads to the same conclusion as that of Kynch (although the reasoning in reaching the conclusion is not the same) so that questions of the validity of the Kynch results are avoided. Moreover, Region 1 is the simplest case, but still allows Font's central result to be demonstrated.

[There is also the question of transient compression resistance in the present "no-compression" concentration range (Dixon, 1978). The effect of this on Region 1 settling is to produce a nonexpanding graded-concentration zone at the suspension/sediment interface. However, assuming the transient compression resistance to be small, this zone will be very shallow. It can be neglected without affecting the remainder of the argument.]

Consider the familiar settling-flux vs. concentration plot, as in Figure 1. The "no-compression" line shows the settling flux in the absence of a compressive stress gradient as a function of concentration; the product of the concentration and the terminal settling velocity. The latter is the settling velocity at which the submerged solids weight is balanced by the liquid

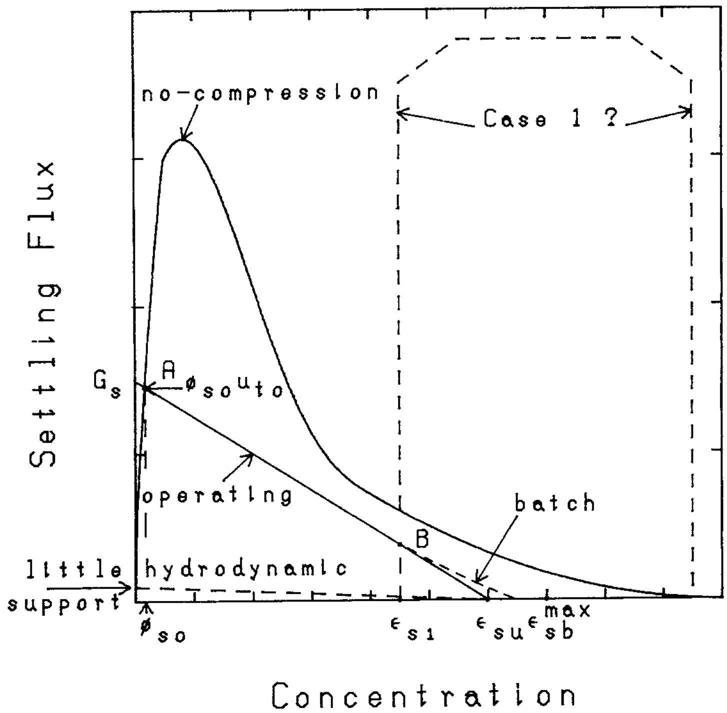

Figure 1. Settling Flux versus Concentration.

drag and is related to the permeability of the solids assembly by Eq. 21 in the paper. In the sediment, the actual settling velocity is less than the terminal (no-compression) value because of the retarding effect of the stress gradient. [It is noted that "settling" velocities and fluxes are measured relative to the bulk (volume-average) velocity of the solid/liquid mixture.]

Consider batch settling, with initial concentration ϕ_{so} in Region 1. The solids in the suspension rapidly accelerate to the terminal velocity $u_t(\phi_{so}) = u_{to}$, say, corresponding to the initial concentration. Because ϕ_{so} is in Region 1, the point (ϕ_{so}, $\phi_{so}\underline{u}_{to}$) on the no-compression line (point A in Figure 1) is below the tangent from the maximum-concentration point on the concentration axis to the underside of the no-compression line. Again because ϕ_{so} is in Region 1, no concentration gradient develops in the suspension, either by $u = u(c)$ arguments or the writer's arguments. Thus, the concentration remains uniform at ϕ_{so}, containing solids settling at u_{to}, but diminishes in depth as solids arrive at the sediment. As the suspension diminishes, the sediment increases. The concentration remains at ϵ_{s1} at the top but is progressively compressed to higher values at the bottom and at intermediate depths, as the effective weight above increases.

Consider some instant t_{1d} (Figure 7 in the paper) during the batch settling process, at which the sediment depth is observed to be L_{1d} and the rate of increase dL_1/dt. At the suspension/sediment interface, there is a jump in solids settling flux from $\phi_{so}u_{to}$ to $\epsilon_{s1}u_{s1}$, accompanied by a jump in concentration from ϕ_{so} to ϵ_{s1}. The well-known expression for the velocity of a discontinuity, as used by Kynch and derived from the solids material balance across the discontinuity, is

$$u_I = \frac{\phi_{so}u_{to} - \epsilon_{s1}u_{s1}}{\phi_{so} - \epsilon_{s1}} \qquad (1)$$

in which all velocities are defined as positive downward. u_I is the velocity of the discontinuity relative to the bulk velocity and is normally negative. In a batch process, the bulk velocity is zero relative to the container, so that $dL_1/dT = -u_I$.

Now, in steady-state, continuous thickening starting from ϕ_{so}, there will be a discontinuity between concentrations ϕ_{so} and ϵ_{s1} at the suspension/sediment interface, and the movement of this interface relative to the bulk velocity will also follow Eq. 1. (Of course, u_{s1}, the solids settling velocity at the top of the sediment, will generally be different from u_{s1} at an arbitrary time during batch settling with the same ϕ_{so}.) In the steady-state, continuous case, the interface will be stationary relative to the equipment, but still moving relative to the bulk velocity in accord with Eq. 1. In continuous operation, the bulk velocity, v_u^c, is determined by the sludge pumping rate. Thus, $v_u^c = -u_I$.

[In referring to steady-state operation starting from ϕ_{so}, it is implied that the conditions in the clarification zone of the thickener are not being considered. It is assumed that there is sufficient depth for clarification to allow the assumption that the feed solids spread to cover the whole cross section at concentration ϕ_{so} before reaching the sediment (Dixon, 1985, 1988).]

The continuous-operation case that corresponds to the batch situation at time t_{1d} is defined as that for which v_u^c equals dL_1/dt. The straight operating line for this case, under the sludge plug-flow assumption, passes through point A with slope $-v_u^c$ and shows how the settling flux must vary with concentration through the thickener. The operating line is derived from the solids material balance. For example, the concentration-axis intercept is the underflow concentration, ϵ_{su}^c, and the ordinate-axis intercepts the solids volumetric throughput rate per unit area, G_s. What we wish to deduce is the relation between the sediment depth in the batch test at time t_{1d} and the depth required for the corresponding continuous operation defined. To do this, the instantaneous $\epsilon_s u_s$–ϵ_s variation in the batch test (the "instantaneous batch operating line") is compared with the linear variation in the corresponding steady-state, continuous case.

First, the batch line agrees with the continuous line in the ϕ_{so}–ϵ_{s1} interval, that is, across the suspension/sediment interface. The slope is given by the righthand side of Eq. 1, which is the same for both by definition. Thus, the settling flux at the top of the sediment in the comparative continuous case is the same as in the batch case (point B in Figure 1). Since the remainder of the line (through the sediment concentration range) will be concave upwards, it must lie above the continuous line, leading to a bottom concentration which is larger than that in the continuous case.

This follows from the argument given by Font in reference to his Figure 6. The further a concentration is from the bottom, the smaller the compressive effect is expected to be. Thus, the settling velocity is closer to the terminal value, and there is less unsupported weight to contribute to sediment compression and thus smaller rate of change of concentration with depth (cf. Dixon, 1982). Therefore, the lines of constant concentration move further apart as the sediment builds up: that is, smaller concentrations are moving upward more rapidly than the larger ones.

The velocity of movemnt, u_c, of a plane of fixed concentration is given by the equivalent of Eq. 1 for regions of continuous variation,

$$u_c = \left[\frac{\partial(\epsilon_s u_s)}{\partial \epsilon_s}\right]_t \tag{2}$$

equivalent to Font's Eq. 30. Since smaller concentrations are moving faster than the larger ones, at any instant $d(\epsilon_s u_s)/d\epsilon_s$ decreases in absolute value with increase in concentration, so that the batch operating line is concave upward. It then follows immediately from Eq. 32 or 33 (which is identical if u_s and u_s^c are both referred to the bulk velocity) that the sediment depth is larger in the batch case than the continuous. Hence, the batch depth provides an overestimate of the depth required for the corresponding continuous case, which is Font's result.

In relation to Eqs. 32 and 33, it will be seen that in both cases the denominator of the integrand at any concentration is proportional to the vertical distance between the operating and no-compression lines. Thus, the closer the operating and no-compression lines, the greater the sediment depth required. Thus, since the instantaneous batch operating line lies closer to the no-compression line, the sediment depth observed at t_{1d} is greater than that in the corresponding continuous operation. The nearer the operating line is to the no-compression line at a given concentration, the greater the proportion of the solids submerged weight that is supported hydrodynamically (rather than mechanically) at that concentration.

For a given batch test, observations of L_1 and dL_1/dt can be made at a succession of times, giving estimates of the continuous depths required for a pencil of operating lines, of successively smaller slopes, all passing through point A. Given the existence of a maximum concentration, the batch operating line will become straight at the time when $\epsilon_{sb} = \epsilon_{sb}^{\max}$, as in the paper. The batch line will then coincide with the comparison continuous line, ϵ_{su}^c will equal $\epsilon_{sb}^{\max}$, and the batch depth will equal the continuous depth. This marks the end of the data that can be extracted from the particular test, where dL_1/dt becomes effectively constant.

By using a higher initial concentration in the batch test, the limiting operating line can be made to approach tangency with the no-compression line, while remaining in Region 1. Initial concentrations beyond the Region 1 limit will also provide data in a similar way; however, as noted above, there are further complicating factors, which are avoided for the present, for the sake of brevity and clarity. While the author might not agree, the above analysis based on ϕ_{so} in Region 1 shows the essence of Font's new concept.

[The author might protest that it is impossible for a thickener to have an operating line that is not tangential to the no-compression line; at least when the point of tangency is in the no-compression concentration range. In other words, there is only one solids throughput rate, G_s, for a given underflow concentration ϵ_{su}^c. However, a statement equivalent to this would be considered very strange if made in connection with other operations, such as distillation and heat exchange. If a certain ϵ_{su}^c can be achieved at a certain G_s, the same ϵ_{su}^c can certainly be achieved at a lesser value of G_s. In other words, if G_s and v_u^c are reduced in equal proportions, in order to rotate the operating line anticlockwise around the ϵ_{su}^c point on the concentration axis (cf. Dixon, 1988), the same ϵ_{su}^c can be achieved.]

It will be noticed that there is no mention of Font's Case 1 in the above development, whereas Case 1 plays a central role in the treatment given in the paper. In this case, hydrodynamic support of the solids is assumed to be negligible compared to mechanical support in the compression zone. To see the implications of this assumption, consider first the continuous case.

If hydrodynamic support is negligible, the operating line is nearly horizontal, as indicated in Figure 1 and in accordance with the observations above on the relation between operating and no-compression lines. Thus, the throughput rate is very low. The only alternative to this interpretation, if it is only the compression range in which the hydrodynamic support is low, is that the no-compression line has a large hump in the ϵ_{s1} to $\epsilon_{sb}^{\max}$ range, as shown by the dashed lines in Figure 1. This hardly seems likely in any real solid-liquid system.

With a nearly horizontal operating line, the sludge in the continuous thickener is moving downwards very slowly with each layer supported almost entirely by mechanical interaction from the layer below. In the zero throughput limit, the concentration variation with depth will be the same as in the sludge at the completion of batch settling. It follows immediately by inspection of Fitch's equation (Eq. 33 in the present paper), as noted previously by the writer (1980, 1985), and corresponds with statements a and b in the Case 1 section of the present paper.

The difficulty with Font's treatment of Case 1 is that the operating line is shown as close (tangential, in fact) to the no-compression line, leading to an area requirement (G_s upper limit), which can only be meaningful if the hump in Figure 1 occurs. Given any reasonable assumption on the shape of the no-compression line, negligible hydrodynamic support corresponds to the zero throughput limit.

In the case of batch settling, for there to be negligible hydrodynamic support in the sediment other than at the completion of settling, the passage rate of solids between suspension and sediment must be small unless the "hump" is assumed to be present in the no-compression line. When this is the case, the consolidation of the sediment is virtually complete at each instant. For the build-up rate of sediment to be small, ϕ_{so} must be small so that $\phi_{so}u_{to}$ is small. The batch conditions then correspond to those of the continuous operating line for little hydrodynamic support shown in Figure 1.

The difficulty with Case 1 is passed on to the treatment of Case 2, which is the general case discussed above. In considering the relation between the sludge depth at time t_{1d} and the corresponding continuous depth, the author compares with a "hypothetical situation," in which there is negligible hydrodynamic support in the compression range. In this case, it is definitely implied that the no-compression curve does have a hump. While this is not likely to occur in practice, anything is allowable in a hypothetical situation. However, difficulties persist. In reference to Figure 7, it is deduced that the concentration at the bottom at time t_{1d} in the hypothetical case is less than that in the real case. In the hypothetical case, however, hydrodynamic solids support is negligible, so that the whole of the submerged weight of each layer contributes to the compressive stress below. Thus, for the same depth in real and hypothetical cases the bottom concentration must be higher in the hypothetical case. If either is correct, it must be the curved operating line in Figure 8 that is the hypothetical case and the straight the real one; but, the other way around is what is required. The references to Case 1 serve only to cloud the argument. The essential final conclusion is correct as discussed above, but it is not in any way related to Case 1. In Figure 8, the two operating lines are the same shape as those shown in the writer's Figure 1. However, the curved line is the real batch line, and the straight one is the line in the corresponding continuous case. The correspondence between the batch and continuous cases is established by equating the operating lines over the ϕ_{so} to ϵ_{sl} range (points A to B), as discussed above.

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Reply:

I would like to thank Dr. Dixon for his interest in my work and for taking time to clarify some points of my paper.

His letter starts with some comments on Dr. Dixon's useful contributions to the sedimentation theory: funneling effects, no-compression zone and compression zone, and transient compression resistance. These aspects are not considered important by Dr. Dixon, so I will not discuss their convenience, utility or validity. In my paper, the fundamentals of the method were presented. The work is based on the Kynch theorems, developed previously by Dr. Fitch and Dr. Tiller. These theorems are applied only to the no-compression zone in batch testing and in continuous thickeners. Dr. Dixon's papers present an analysis of the sedimentation based on assumptions different from those considered in my paper. Certainly, constructive discussions among researches in this area and a comparison of the results predicted by different theories with the experimental results can contribute to better understanding of the sedimentation and its application to the design of thickeners.

Dr. Dixon accepts the conclusions drawn in my paper, but disagrees with the procedure for obtaining them. In his letter, he presents an interesting analysis of a particular case, when ϕ_{so} is in Region 1 in Figure 1 of his letter. I consider his argument to be correct except for the following points:

- In some cases, such as that considered in Dr. Dixon's letter (ϕ_{so} in Region 1), the operating line intercepts the flux density curve and is not tangent to the curve. This can also be considered a consequence of the Kynch theorems applied to the no-compression zone. Dr. Fitch has extensively analyzed the rising of characteristics and discontinuities in his papers (as cited in the Literature Cited section of my paper).
- It is possible to design a continuous thickener as indicated in Figure 1 of this reply. In this case, there is a concentration discontinuity between points A and B (Figure 1). The rising velocity of this discontinuity (absolute value of the slope corresponding to the straight line which passes by points A and B) is greater than the downward velocity of pulp resulting from underflow withdrawal (absolute value of the operating line B-C). The latter is a consequence of the consideration of Eq. 1 in Dr. Dixon's letter and the

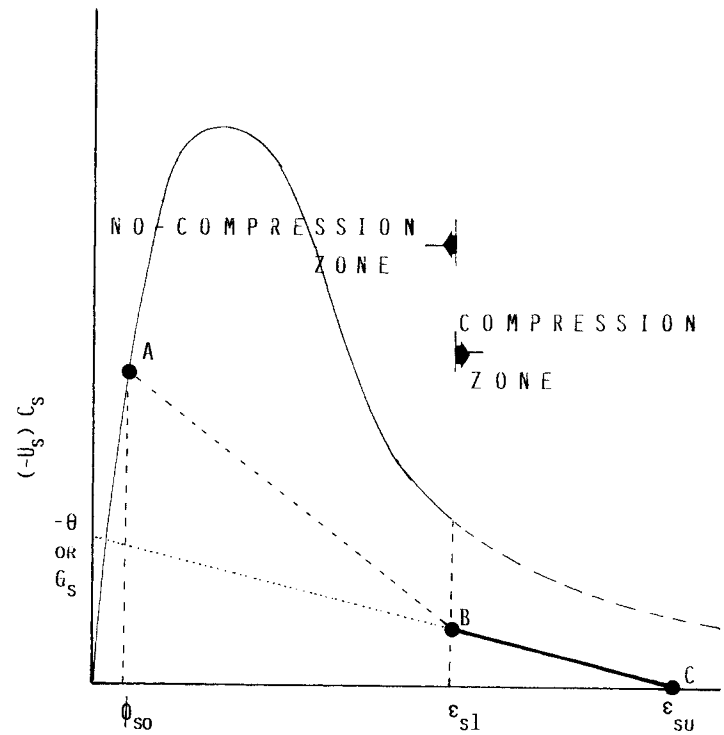

Figure 1. Yoshioka construction with the maximum value of G_s.

concept of the operating line. That means that the solids, when entering in the thickener, must reach the concentration ϵ_{s1} in a small transient zone, because any layer with concentration ϕ_{so} or any intermediate value between ϕ_{so} and ϵ_{s1} cannot remain above the sediment for a long period of time.

The proposed design by the Yoshioka construction is shown in Figure 2. In this case, there is a discontinuity between the points A and B that moves with the same velocity as the downward velocity of pulp resulting from underflow withdrawal, but in the opposite direction. It means that a zone with concentration ϕ_{so} can remain above the sediment in the continuous thickener. The design of the continuous thickener in accordance with Figure 2 presents two advantages with respect to Figure 1 of this reply:

- The thickener volumetric flux density, θ (or G_s) is greater in Figure 2 than that in Figure 1. In Figure 2, the operating line is closer to the terminal settling flux density vs. concentration curve than that in Figure 1. The compression zone height in a continuous thickener must, therefore, be greater in the case of Figure 2 than in the case of Figure 1. From batch testing, the continuous thickener design can be carried out according to Figure 2 with small zone compression height (although greater than that in Figure 1). Consequently, it is more convenient to design the continuous thickener with the highest possible value of G_s.
- Any fluctuation in the inlet flow rate would affect the amount of solids in the sediment and their corresponding capacity to compress lower layers in the case of Figure 1. This is due to the fact that there is no stable layer above the sediment, and there is only the amount of solids corresponding to the transient zone between ϕ_{so} and ϵ_{s1}. In the case of Figure 2, above the sediment, the stable zone with concentration ϕ_{so} can absorb fluctuations in the inlet flow rate, without altering the compression zone depth. Of course, the previous advantage is more important than this one.

Consequently, I consider that the design of the continuous thickener must be done as indicated in Figure 2 in the case discussed (ϕ_{so} in Region 1 in Figure 1 of the letter). In other cases, as presented in my paper, the operating line must be tangent to the settling flux curve in the no-compression range. In these cases, there is no analogy between the thickener

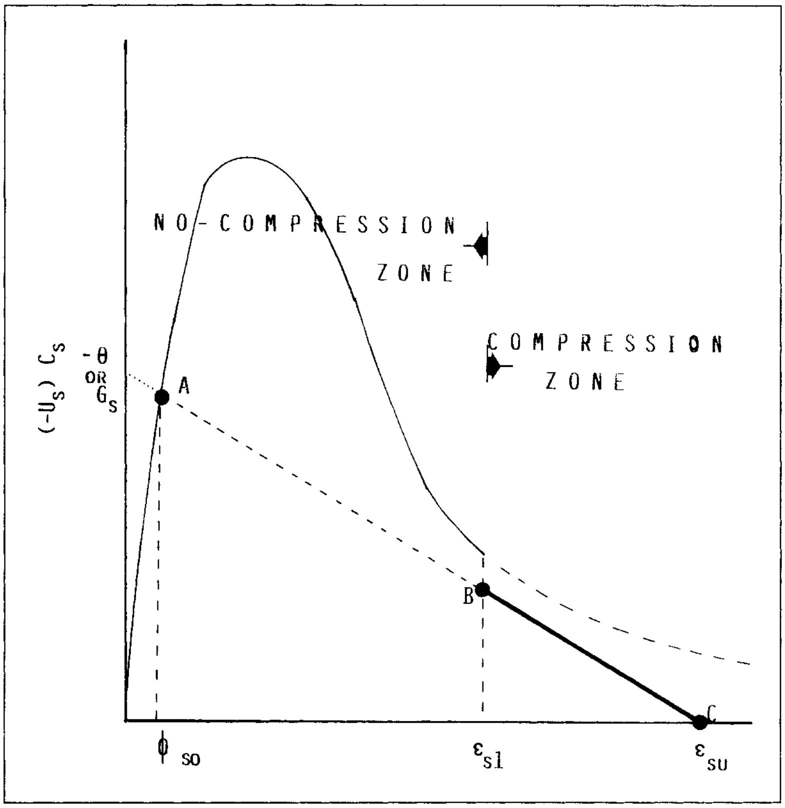

Figure 2. Possible case.

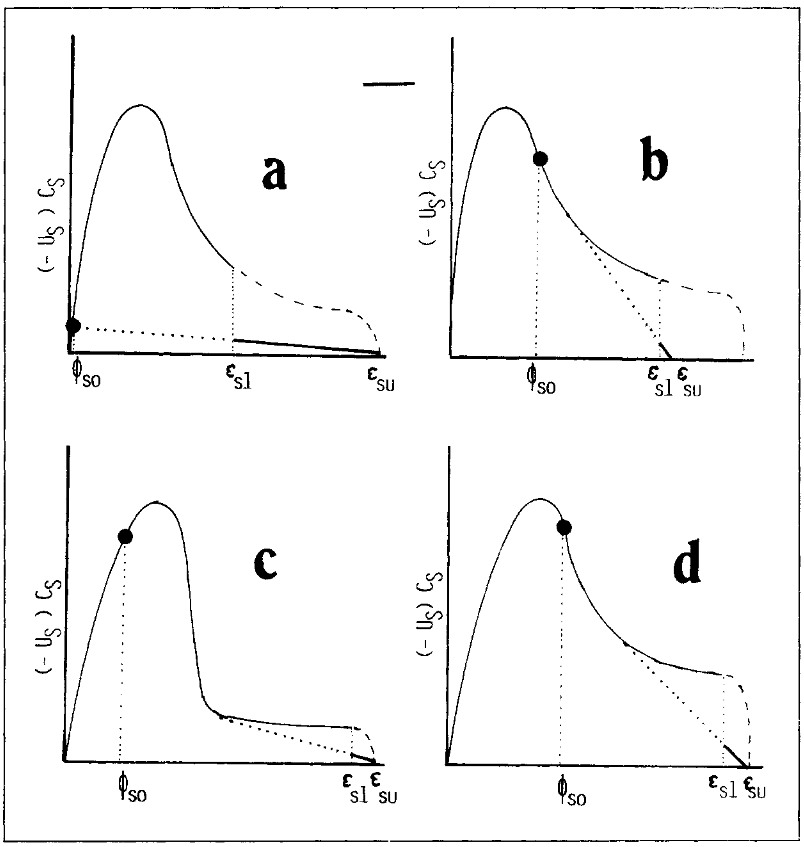

Figure 3. Other alternatives.

and a distillation column or a heat exchanger.

Other very distinct situations take place when the initial solids concentration is in the compression range or when, although the initial solids concentration is inside the no-compression range, the operating line approaches the terminal settling flux curve in the compression range. In these cases, there is an analogy between the continuous thickener and a column distillation with respect to the concepts of operative line and equilibrium line (terminal settling flux curve in continuous thickeners).

As far as the comments on Case I are concerned, I would like to mention the following:

• The analysis of Case I is presented mainly to obtain the conclusions in the general Case II.

• With respect to the "hump" in the settling flux density curve, I consider that the discussion presented by Dr. Dixon is correct, but is incomplete. The title of the section, where Case I is described in my paper, is "Case I: very high permeability $[(dp_s/d\epsilon_s)(\delta\epsilon_s(z)/\delta z) \gg (\mu/k)(u_s)]$." Under this heading, it is assumed that the values k of permeability are high enough to accept the inequality presented. This means that the operating line is far from the terminal settling flux density vs. concentration curve. Dr. Dixon proposes a possible case, where the terminal settling flux density vs. concentration curve has a "hump." Other cases can be considered as shown in Figure 3.

With normal suspensions, whose hydrodynamics are close to that proposed by the Michaels and Bolger equation in the no-compression range, Case I cannot occur, except when the initial solids concentration ϕ_{so} is very low (Case a in Figure 3) or when the underflow solids concentration ϵ_{su} is close to the value ϵ_{s1}—limit between compression range and no-compression range (Cases b and d in Figure 3). With these normal suspensions, the hypothetical and fictional Case I (with the corresponding "hump") is only considered to obtain the conclusions stated in Case II.

• In the analysis presented in Case II, two suspensions with different hydrodynamic and mechanic behavior are considered: the real one and another hypothetical suspension. It is assumed that, using the hypothetical or fictional suspension, the same variations of the discontinuities (supernatant-suspension, sediment surface) heights vs. time are obtained as those corresponding to the real suspension. In this hypothetical suspension, it is assumed that the hydrodynamic solid support is negligible with respect to the solids compression in the compression range under the conditions indicated by the inequation shown for describing Case I.

I deduce that Dr. Dixon considers that the hypothetical suspension has the same relation $p_s = f(\epsilon_s)$ as the real one but with negligible hydrodynamic support (with a "hump" in the settling flux density curve). The latter is very different from that stated in my paper. Dr. Dixon's analysis could be correct, but cannot be applied to the discussion presented in my paper to obtain the corresponding conclusions.

• The consideration of Case I is necessary in the analysis of Case II to deduce that the concentration at the bottom of the cylinder in batch testing is greater than the corresponding value with the hypothetical suspension calculated by Eq. 36 (substituting ϵ_{su} by ϵ_{sb}, ϕ_{s2} equals ϕ_{so} and $-u_{s2}$ equals $-u_{so}$ in the situation discussed in the letter). In Dr. Dixon's letter, I have not been able to see an alternative method to deduce this question. In any case, I thank Dr. Dixon for his interest in completing or clarifying this question of arriving at the same conclusion.

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